

# **DIRECT**

## Operational Field Test Evaluation Simulation and Modeling

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# **DIRECT Simulation: Benefits of Traffic Information**

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## **Abstract**

The purpose of the simulation evaluation is to assess the expected future impacts of the DIRECT technologies under scenarios of full deployment. This provided some indication of the level of benefits that can be expected from DIRECT in the future. Because deployment of some of the DIRECT communications systems required purchase of special receivers for the vehicle, and installation of expensive roadside communications devices along the included corridors, full deployment (i.e., all vehicles and all corridors) was expensive to test in the field

The simulation was designed to compare the travel times of DIRECT-equipped vehicles against the travel times of unequipped vehicles. The equipped vehicles were about 25 percent of the total vehicles in the incident area.

1. Delay Period: This is the time from when an incident occurs until the intelligent vehicle is informed about it. The delay period was 20 minutes for all the runs.
2. Incident Location and Duration: This parameter specifies where the incident is located and for how long it is present. Three incident locations were analyzed, one on I75 in Troy, another on Rochester Road near I75, and the last was on I696 near I75. The incidents blocked traffic moving into Detroit during the morning rush hour. The locations are shown on a map in Figure 1.

The findings show what might be expected when traffic information is provided to drivers encountering short traffic incidents in the range of five to 10 minutes. When the incident occurs in a location where there are easy and efficient alternative routes, and the drivers that have access to traffic information from a 20 minute periodic report, the drivers using the traffic information systems experience a short average delay of around four minutes, while the drivers without the information experience a much longer delay of over 16 minutes. The DIRECT survey of drivers indicates that a delay over 12 minutes is not tolerable for most commuters. Under these conditions the driver receiving traffic information is likely to be much more satisfied with the commute than the driver that does not have access to this information.

Traffic information helps travelers get to their destinations faster even when there are fewer alternative routes. However, as the alternatives diminish to do the travel time benefits. Under conditions where there are few good alternative routes and the travel time benefits disappear the informed traveler may experience the additional benefit of knowing why there is a delay and how long it might last. This type of psychological benefit is addressed in the driver survey report, and it is possible that knowing about an incident offers even greater value to the driver than travel time benefits. Nevertheless, the personal travel time benefits of traffic information are clear and quantifiable as long as there are reasonable alternative routes to the destination.

## Contents

1. INTRODUCTION AND OBJECTIVES.....	4
2. LITERATURE REVIEW .....	4
3. SIMULATION APPROACH.....	5
4. TRAFFIC SIMULATION.....	6
4.1 CALIBRATION . . . . .	8
5. DATA REQUIREMENTS-.....	9
5.1 TRAFFIC NETWORK CONFIGURATION .....	10
5.2 TRAFFIC VOLUMES AND TRIP DATA .....	11
6. SIMULATION SCENARIOS .....	12
7. SIMULATION RESULTS AND ANALYSIS .....	13
8. CONCLUSION.....	14

## **1. Introduction and Objectives**

The DIRECT project focuses on selecting near-term available methods that show promise of improving present delivery methods of traffic information to motorists. There are four methods compared in DIRECT: (1) low power highway advisory radio (LPHAR), (2) automatic highway advisory radio (AHAR), (3) cellular call-in, and (4) Radio Broadcast Data System (RBDS).

The purpose of the simulation evaluation is to assess the expected future impacts of the DIRECT technologies under scenarios of full deployment. This provided some indication of the level of benefits that can be expected from DIRECT in the future. Because deployment of some of the DIRECT communications systems required purchase of special receivers for the vehicle, and installation of expensive roadside communications devices along the included corridors, full deployment (i.e., all vehicles and all corridors) was expensive to test in the field

The traffic simulation model developed at the University of Michigan has the ability to represent the routing of individual vehicles dynamically in a traffic network and their response to various types of information systems. When vehicles change their routes in response to information they also change the overall traffic and link flows in the network. This ability to represent individual vehicles and their routes in response to information provides a mechanism for assessing the impact of motorist information on individual vehicles and on traffic in general.

The UM simulation model also provides for multiple vehicle types, where the definition of type is based on the routing logic of the vehicle. So, a specified proportion of vehicles can be designated as “background” vehicles and their routes would be determined without the benefit of real-time traffic information. Other vehicles can be designated as “intelligent” vehicles and their routes are determined based on the time and location at which the information it is received, as well as the behavior of the driver. The market penetration analysis consists of conducting an impact assessment of progressively increasing the proportion of “intelligent” vehicles.

## **2. Literature Review**

One of the first simulation-based evaluations of the benefits of in-vehicle information systems was performed by Al-Deek, et al. (1988) for the PATH program at the University of California at Berkeley. The simulation looked at the potential benefits of in-vehicle information systems in a real-life freeway corridor under recurring and incident-induced congestion. This simulation study employed FREQ8PC to represent the freeways and TRANSYT-7F to represent the arterials. The simulation modeled traffic along the Santa Monica Freeway corridor. They assumed that informed drivers would take the static shortest route. The results were that

under recurring, non-incident congestion, the travel-time savings were negligible (i.e., no greater than 3 minutes for a 25 minute trip). However, under the non-recurring, incident congestion scenario, travel time savings were found to be significant (i.e., greater than 3 minutes). However, this research has been criticized for combining dissimilar models to represent the freeways and arterials.

### **3. Simulation Approach**

The University of Michigan is using the TRAFFIC simulation model to represent and analyze the DIRECT vehicles. The main characteristics of this model is that it abstracts away much of the link and intersection detail, as well as the inter-vehicle dynamics on links and at intersection, in favor of capturing the global network dynamics of traffic responding to changes in traffic demand interacting with an integrated freeway/traffic signal network.

This simulation has the following capabilities, which are particularly important for evaluating traveler information systems:

- Ability to define a range of simultaneous routing strategies for vehicles entering the network (e.g., drivers with different levels of real-time information and background router for the control group),
- periodic update of routing matrices,
- Traffic information and driver route selection interaction,
  - modeling peak period congestion effects,
  - incident modeling (as temporary link speed reduction or lane blockage), and
  - individual vehicle tracking.

There are several reasons for using a route-based network simulation for evaluating DIRECT. First, this type of simulation enables the evaluation of large numbers of vehicles using various routing strategies. Second, the computational efficiency of the event-based program structure, and the simple queueing-based traffic representation, enables a sufficiently large network to be modeled for analysis of guidance and rerouting over realistic commute distances. Third, the simulation provides the mechanism for analyzing the interaction between traffic control and route guidance. The co-location of the DIRECT highway commutes and surface street signalization in case of an incident is crucial.

This study uses the existing University of Michigan TRAFFIC simulation model. This model has the capability of:

- Modeling interaction of freeways and signalized arterials
- Modeling peak period congestion effects
- Real-time control and route guidance interaction

- Ability to define a range of simultaneous routing schemes for vehicles entering the network.

The model uses impedance functions and queues to move vehicles on the links. Due to the different characteristics of traffic flow that need to be modeled on freeways and at traffic signals, it analyzes traffic flow in terms of vehicles as individual entities. This approach permits a traffic flow representation common to both freeways and urban streets. Furthermore, it permits a continuous dynamic queuing-based traffic assignment. The common traffic flow representation is critical to modeling all network components in a consistent and compatible fashion, while the queuing-based dynamic traffic assignment technique is essential to dealing with diversion and re-routing of traffic during congestion and in response to any incidents.

The consideration of individual vehicles is primarily for purposes of improving the analysis resolution during the internal calculations, and does not necessarily require the user to collect or input data at the individual vehicle level. Instead, traffic flow characteristics and traffic demands can be specified by the user at an aggregate level (e.g., departure rates instead of exact specification of vehicles' departures), leaving it to the model routines to derive the measures related to individual vehicles.

The simulation emphasizes the time-dependent routing of individual vehicles through the Detroit Metro area. It assigns vehicles to routes connecting pre-assigned origin-destination pairs in accordance with a departure rate and a specified routing strategy. As time passes, vehicles enter the network at their origin, travel a specified path, and terminate the trip when the destination is reached. Vehicles enter the network over time as specified by the departure rate distribution. The progress to the destination is influenced by a variety of factors including link capacities, congestion levels, traffic signals, and incidents. As in a real road network, a vehicle is slowed down by congestion that is caused by other vehicles on the network. This congestion could take the form of link impedances and/or various forms of queuing.

#### **4. TRAFFIC Simulation**

We propose to use the TRAFFIC model developed at the University of Michigan. The TRAFFIC simulation was inspired by the INTEGRATION simulation developed by Mike Van Aerde of Queens University, which the ITS Modeling and Simulation Research Group at the University of Michigan (UM) has extended and enhanced. The University of Michigan's use of INTEGRATION was a result of a cooperative research arrangement between UM, MTO and Mike Van Aerde. The Michigan research group originally acquired the INTEGRATION simulation to test route guidance algorithms under development at UM. Then, the University of Michigan became involved in a more comprehensive simulation effort for the Detroit Metro area using the TRAFFIC simulation.

TRAFFIC is a unique traffic evaluation tool because it helps define multiple vehicle types which may employ alternative routing schemes. The individual vehicles move through the defined network according to defined schemes. The paths and interrelationships between the vehicle behaviors can be evaluated within the context of the model.

TRAFFIC was developed specifically to evaluate and optimize the operation of integrated freeway/traffic signal networks during periods of recurring and non-recurring congestion. The motivation for developing TRAFFIC can be summarized as follows:

- Need to predict future/alternative network behavior
- Desire to evaluate potential user/system benefits
- Need to pre-evaluate and test new control strategies
- Effort to optimize network performance

The approach considers the behavior of traffic flow in terms of individual vehicles that have self-assignment capabilities. This capability serves as a traffic assignment function and circumvents the need to use either an explicit time slice or iterations during the traffic assignment. Consequently, one can consider continuously variable traffic demands and controls, both freeway and signalized networks, as well as any links that join them. TRAFFIC has the following unique capabilities which make it appropriate for evaluating route guidance and driver information systems:

- Modeling interaction of freeways and signalized arterials
- Modeling peak period congestion effects
- Real-time control and route guidance interaction
- Ability to define a range of simultaneous routing schemes for vehicles entering the network.

The capabilities and characteristics of the basic TRAFFIC simulation approach are reviewed in several published papers. The list below highlights several of these capabilities.

- Microscopic model of individual vehicle movements
- Trip Origin/Destination and departure time is fixed
- Actual route is selected en-route based on routing vectors
- Routing vectors are updated periodically
- Vehicles are queued at traffic signals during red signal phases
- Vehicles are released at saturation flow rate during green phases
- Signal timings are updated based on on-line traffic flows



- Incidents are modeled as temporary lane blockages
- Routings can respond to controls and congestion

Due to different characteristics of traffic flow that need to be modeled on freeways and at traffic signals, TRAFFIC analyzes traffic flows in terms of vehicles as individual entities. This microscopic approach permits a traffic flow representation which is not only common to both types of component networks, but also permits a continuous dynamic queuing-based traffic assignment. The common traffic flow representation is critical to modeling all network components in a consistent and compatible fashion, while the queuing-based dynamic traffic assignment technique is essential to dealing with diversion and re-routing of traffic during congestion and in response to any incidents.

The model's consideration of individual vehicles is primarily for purposes of improving the analysis resolution during the model's internal calculations, and does not necessarily require the user to collect or input data at the individual vehicle level. Instead, traffic flow characteristics and traffic demands can be specified by the user at an aggregate level, leaving it to the model routines to derive the more microscopic measures.

#### **4.1 Calibration**

The simulation was be calibrated in two phases:

##### **Phase 1**

To calibrate TRAFFIC for the DIRECT network to evaluate:

1. Likely benefits of improved en-route driver information
2. Alternative options for system configuration
3. Methods for implementing a real-time control scheme

##### **Phase 2**

To compare the simulation findings to actual DIRECT field data to:

1. Determine the degree of success of the simulation model and to make any further model improvements
2. Determine how the DIRECT results could be used to estimate the field performance of ATIS in other cities

Vehilces in the DIRECT control group are generally driven by commuters familiar with the route during the peak period. Commuters minimize their trip time with relatively little navigational error. Typically, these drivers anticipate recurring traffic and make routing adjustments over time to minimize their overall travel time. In addition, background vehicles are slower to respond to incidents and to recurrent congestion than guided vehicles.

This simulation model uses a combination of route assignment methods to represent background traffic during the morning commute. The goal is to achieve a realistic temporal and spatial distribution of vehicles while representing different types of travel behavior. Typically, these drivers anticipate recurring traffic and make routing adjustments over time to minimize their overall travel time. In addition, background vehicles are slower to respond to incidents and to recurrent congestion than guided vehicles. Two routing strategies are implemented to represent the background traffic: anticipatory-based route assignment and real-time routing with path archiving.

On a given day, drivers who are familiar with network conditions should be able to make efficient pre-trip route choices. Typically, these drivers are influenced by historical perceptions of travel time. Hence, they anticipate recurring traffic and make routing adjustments over time to minimize their overall trip time. In this model, routes assigned to drivers of this class are computed using a lookahead shortest path algorithm. This algorithm uses travel times that reflect current as well as anticipated traffic conditions, and is applied iteratively to more closely represent the way drivers adjust their perception and knowledge of traffic conditions over time. However, drivers of this class follow their initial routes until the end of their trips and are slow in responding to incidents.

This routing strategy represents the stochastic and complex nature of a group of drivers. This group may include unfamiliar drivers who stay on their initial path until they reach their destinations, and those who update their paths based on perceived current traffic conditions. Paths assigned to these drivers are shortest paths computed using travel times available at the time of computation. Periodically (every 20 **minutes**), a new set of paths is computed, and a random subset of drivers switches to these new paths. The remaining drivers stay on their current paths (referred to as “archived”). The link travel times are estimated based on the experience of all background vehicles. These estimates are generated using the exponential smoothing model and a smoothing factor. This model achieves a realistic distribution of traffic and captures some of the dynamics of congested traffic behavior, at no added computational complexity.

Although two distinct classes of drivers are used to model the background traffic, the following simulation analysis reports only on the overall measures of effectiveness (i.e., averaged over the drivers of both classes). These measures of effectiveness are referred to being related to *unguided* (or background) vehicles. An in-depth study of the behavior of these two classes is beyond the scope of this evaluation effort.

## **5. Data Requirements:**

The simulation was tested for the I-75 corridor. This required the collection of 5 to 15 minute traffic counts for all streets in the area. These counts were collected via

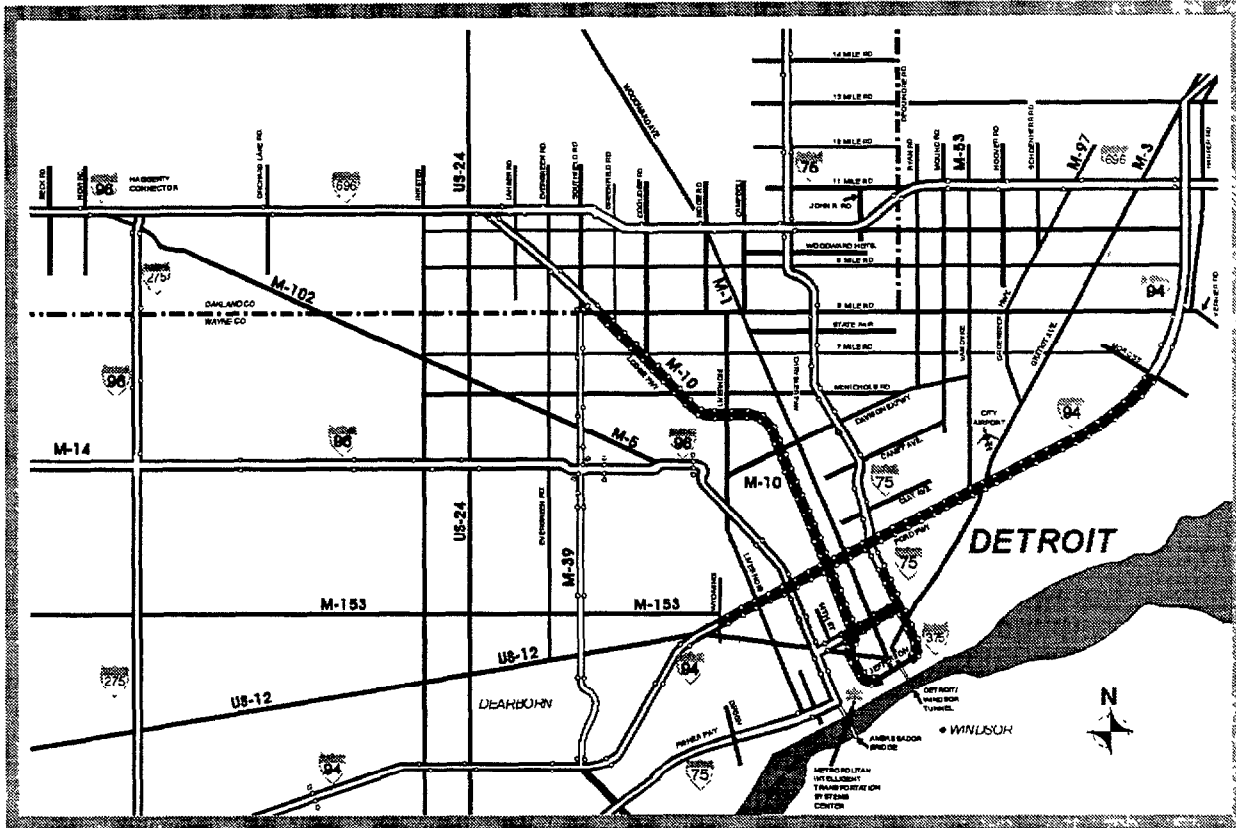
loops over a period of about a week. Inputs to the model include: (1) network geometry, (2) link characteristics, (3) traffic signal timing, (4) dynamic traffic demand, and (5) routing heuristics for background and “informed” drivers. Dynamic traffic demand was derived from the 15 minute counts using a dynamic synthetic O-D technique.

There are two types of data required to run the simulation scenarios. First the data that represents the roadway networks (i.e., links and nodes and their respective specifications, and traffic signal timing). Second, the trip data for loading the network with vehicles traveling between pre-specified origins-destinations (O-D) pairs. Below is a detailed description of these data.

### **5.1 Traffic Network Configuration**

The traffic network used in the simulation covers a part of the Detroit area surrounding the I-75 freeway. In the DIRECT project, the I-75 was selected as the corridor of interest. Therefore, in order to study the behavior of drivers in case an incident occurs on this freeway, other surrounding alternative routes (such as Woodward and Telegraph) were also included in the simulation network. Figure 1 depicts this DIRECT simulation. This network also includes all current signal data for signalized intersections.

Figure 1. DIRECT Network



## 5.2 Traffic Volumes and Trip Data

The simulation emphasizes the time-dependent routing of individual vehicles through the Troy area. It assigns vehicles to routes connecting pre-assigned origin-destination pairs in accordance with a departure rate and a specified routing strategy. As time passes, vehicles enter the network at their origin, travel a specified path, and terminate the trip when the destination is reached. Vehicles enter the network over time as specified by the departure rate distribution. The specific information regarding origin and destination (O-D) locations and their associated departure rate distributions constitutes the trip data. In this simulation study these data have been generated based on SEMCOG's O-D definitions and daily trip data, as well as traffic volumes acquired from MDOT.

The trip data provided by SEMCOG represent average number of trips between O-D. Since the fluctuation of traffic demand would not be captured by these static data, we use a dynamic trip generation approach that provides a more accurate representation of traffic. This approach consists a synthetic origin-destination model coupled with the sequential generation method. In this simulation, we were

interested in the traffic demand during the morning peak hours. Furthermore, the entire peak period is broken down into a series of consecutive time slices (of 15 minutes). The SODGE-based approach results in a separate O-D matrix for each time slice.

The synthetic origin-destination demand estimation approach was based on the Information Minimization algorithm developed by Van Zuylen and Willumsen [3], and revised by Van Zuylen[2]. It required three input files. The first is a network description file. The second file contains link volumes, while the third contains path tree(s). Another optional file is a seed O-D matrix that may be used to assist in initiating the search. In this study, we used the average daily trip data as the seed matrix. One of the measures that are used to monitor and evaluate the convergence is the root mean squared difference (RMSD) between the observed link volumes and the volumes that are produced by feeding the iterated trip matrix back into the network. For this simulation study, the demand estimation approach generated O-D trip data such that when traffic is simulated, the resulting link volumes matched the volumes observed on field. The latter were used as a calibration tool for the simulation.

The above description of the synthetic origin-destination estimation approach for estimating one O-D matrix. In order to generate a sequence of O-D matrices associated with the different time slices, we use the sequential approach. This approach involves the use of previous time slice's O-D matrix as the seed for the derivation of the next slice's matrix. The following is a brief description of the algorithm:

1. Read input data (seed matrix, link volumes, path trees)
2. Used a simple procedure to estimate O-D trip matrix for the desired simulation period (resulting in a seed matrix), and link volumes for each time interval.
3. For each time interval, perform the following:
4. Estimated link volumes from seed matrix and compute link volume error (deviation from the given link volumes),
5. Performed the following iterations until convergence to an insignificant error is reached:
6. Determined O-D correction factor based on link volume error and modify O-D trips,
7. Estimated link volumes from the new O-D matrix and link volume error.

## **6. Simulation Scenarios**

The simulation was designed to compare the travel times of DIRECT-equipped vehicles against the travel times of unequipped vehicles. The equipped vehicles were about 25 percent of the total vehicles in the incident area.

3. **Delay Period:** This is the time from when an incident occurs until the intelligent vehicle is informed about it. The delay period was 20 minutes for all the runs.
4. **Incident Location and Duration:** This parameter specifies where the incident is located and for how long it is present. Three incident locations were analyzed, one on I75 in Troy, another on Rochester Road near I75, and the last was on I696 near I75. The incidents blocked traffic moving into Detroit during the morning rush hour. The locations are shown on a map in Figure 1.

## 7. Simulation Results and Analysis

The three scenarios show that the level of potential travel time improvement from traffic information depends on the location of the incident, the number of good alternative routes, and the information systems that the drivers use. In the first scenario the drivers are east along I696 when the incident occurs. Alternative routes include Woodward Ave. and M10. The unequipped vehicles take an average of 35.09 minutes to drive to their destinations when there are no incidents. The vehicles equipped with the DIRECT information system take about the same amount of time to their destinations. When a short 5 minute delay in traffic occurs eastbound on I696 the vehicles that do not receive traffic information are delayed an average of 16 minutes. Whereas, the vehicles equipped with information systems are delayed less than 4 minutes. This is a significant difference, and it is a result of the many good alternative routes for the drivers traveling along I696. The DIRECT equipped vehicles take a detour around the incident and save about 12 minutes on their trip compared with the unequipped vehicles that are stuck in the short traffic jam. When the incident is increased the unequipped vehicles experience even more delay while the DIRECT vehicle have a short reduction in their travel times.

**Table 1.** DIRECT Compared With Unequipped on I696

(Travel time in minutes)	DIRECT	Unequipped
No Incident	37.09	35.09
5 Minute Incident	40.93	51.31
10 Minute Incident	40.64	52.14

In the next scenario looked at vehicles traveling south along I75 and Rochester Road in the Troy area. In this situation there were few alternative routes for either set of drivers. The DIRECT vehicles traveling in this area had a much shorter baseline travel time of 36.76 minutes compared to 47.50 minutes for the unequipped vehicles. In this case the DIRECT vehicles have more that 10 minutes of benefit even before the incident because they divert around some of the recurrent

congestion in the area. Under the incident conditions the equipped vehicles retain this advantage in the early stages. However, when the incident increases in duration the benefits are reduced because the incident blocks some of the alternative routes that were available in the earlier stages. This scenario shows that the DIRECT vehicles have a greater than 10 minute benefit that shrinks as the incident gets worse.

**Table 2.** DIRECT Compared With Unequipped on Rochester Rd.

(Travel time in minutes)	DIRECT	Unequipped
No Incident	36.76	47.50
5 Minute Incident	40.39	51.31
10 Minute Incident	52.82	56.23

The last scenario the drivers are traveling south on I75 during the peak morning rush. The travel times under the no-incident conditions range from about 40 to 50 minutes because of the distance of the commutes. In this case the benefits of DIRECT are not as great because of the lack of alternative routes. The DIRECT-equipped vehicles have an 8 minute advantage in the standard commute. However, this advantage starts to disappear under incident conditions. With a 5 minute incident on the expressway the unequipped vehicles experience a 7 minute delay because the congestion clears up quickly. However, the DIRECT vehicles experience a delay of more than 10 minutes because the alternative routes that they originally benefited from are blocked and more of the DIRECT drivers stay on the expressway. As a result the DIRECT-equipped vehicles experience 10 to 12 minutes of delay when the incident ranges from 5 to 10 minutes.

**Table 3.** DIRECT Compared With Unequipped on I75

(Travel time in minutes)	DIRECT	Unequipped
No Incident	41.00	49.05
5 Minute Incident	52.74	56.55
10 Minute Incident	52.82	56.23

## 8. Conclusion

The findings show what might be expected when traffic information is provided to drivers encountering short traffic incidents in the range of five to 10 minutes.

When the incident occurs in a location where there are easy and efficient alternative routes, and the drivers that have access to traffic information from a 20 minute periodic report, the drivers using the traffic information systems experience a short average delay of around four minutes, while the drivers without the information experience a much longer delay of over 16 minutes. The DIRECT survey of drivers indicates that a delay over 12 minutes is not tolerable for most commuters. Under these conditions the driver receiving traffic information is likely to be much more satisfied with the commute than the driver that does not have access to this information.

Traffic information helps travelers get to their destinations faster even when there are fewer alternative routes. However, as the alternatives diminish to do the travel time benefits. Under conditions where there are few good alternative routes and the travel time benefits disappear the informed traveler may experience the additional benefit of knowing why there is a delay and how long it might last. This type of psychological benefit is addressed in the driver survey report, and it is possible that knowing about an incident offers even greater value to the driver than travel time benefits. Nevertheless, the personal travel time benefits of traffic information are clear and quantifiable as long as there are reasonable alternative routes to the destination.

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